CSCI 246: Assignment 2
Due: February 18, 2026

Name: ____________________________

Problem 1 (4 points). Let $A = \{1, 2, 3\}$ be a set with three elements.

A. Write all lists containing only elements of A without repitition.

B. Write all subsets of A.

C. How many lists of length k are there that contain only elements of A?

D. What is the cardinality of $\mathcal{P}(A)$?

Problem 2 (6 points). Let $A = \{a, b, c, d, e, f\}$, $B = \{a, e, f, g\}$, and $C = \{e, f, g, h, i\}$.

A. Compute $A \cap B$.

B. Compute $A \cap (B \cup C)$.

C. Are the answers to parts **A** and **B** the same? Give a justification for why or why not.

D. Compute $A - (B \triangle C)$.

E. Compute $A - (B - C)$.

F. Are the answers to parts **D** and **E** the same? Give a justification for why or why not.

Problem 3 (10 points). Let A be a set of n elements. Explain why the number of subsets of cardinality $0 \leq k \leq n$ is exactly:

$$\frac{n!}{(n-k)! \times k!}$$

Hint: Consider the equation for counting the number of lists of length k with no repeated elements and why there are more lists of length k (without reptition) than subsets of cardinality k.

Problem 4 (10 points). Prove the following statement is true:

$$\forall n \in \mathbb{N}. \sum_{k=0}^{n} \frac{n!}{(n-k)! \times k!} = 2^n$$

Hint: Use a counting argument to show that the two sides are equal.

Problem 5 (20 points). For each of the below statements, translate the English statement into a symbolic statement using quantifiers. Clearly define any predicates you introduce (e.g., $prime(n)$ to mean n is prime) and state the domain of each quantified variable. Additionally, for each statement prove or dis-prove the statement (i.e., by proving the negation of the statement if it is false).

A. Every natural number is either even or odd.

B. There is a prime number that is even.

C. Every natural number greater than 1 has a prime divisor.

D. There is a natural number divisible by every natural number.

E. For every natural number n, there exists a natural number m such that $m > n$.

Problem 6 (10 points). Let A and B be disjoint sets. Prove that $\mathcal{P}(A \cup B)$ and $\mathcal{P}(A) \times \mathcal{P}(B)$ have the same cardinality.

Hint: Try showing a bijection between $\mathcal{P}(A \cup B)$ and $\mathcal{P}(A) \times \mathcal{P}(B)$—i.e., that for every subset of the union, there is exactly one element of the product of powersets.