CSCI 246: Assignment 2
Due: February 18, 2026

Name: _____

Problem 1 (5 points). Consider the following relations on $A = \{1, 2, 3, 4\}$:
$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 5)\}$,
$S = \{(1, 1), (1, 3), (2, 2), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4)\}$, and
$T = \{1, 2, 3\} \times \{1, 2, 3\}$.

A. Which of the relations are equivalence relations?

B. Compute $R \cap S$.

C. Is $R \cap S$ an equivalence relation? Explain why or why not.

D. Compute $R \circ S$.

E. Is $R \circ S$ an equivalence relation? Explain why or why not.

Problem 2 (5 points). Let $R = \{1, 2, 3\} \times \{1, 2, 3\} \cup \{(4, 4), (5, 5)\} \cup \{6, 7\} \times \{6, 7\}$ be an equivalence relation on $A = \{1, 2, 3, 4, 5, 6, 7\}$.

A. Write each of the equivalence classes of R.

B. Write the partition P_R induced by R.

C. Determine if the following pairs are in the same part of P_R:

1. 1 and 3.
2. 1 and 6.
3. 7 and 2.
4. 4 and 4.
5. 3 and 2.
6. 5 and 5.

Problem 3 (5 points). Let R and S be equivalence relations on some set A. Prove that $R \cap S$ is an equivalence relation on A.

Problem 4 (5 points). Let R and S be equivalence relations on some set A. Prove that $R \circ S$ is an equivalence relation on A.

Problem 5 (5 points). Let R be an equivalence relation on some set A. Prove that $R \circ R = R$.

Problem 6 (5 points). Prove that for any set A, there are an equal number of partitions of A and equivalence relations on A.

Problem 7 (5 points). Consider bit-strings (i.e., sequences of bits $b \in \{0,1\}^*$)

A. How many 32-bit bit-strings have exactly 5 bits set.

B. How many 32-bit bit-strings have exactly 27 bits set.

C. Are the answers to parts **A** and **B** the same? Explain.

D. Suppose you are sending a block of data containing 128-bits over a faulty networrk connection. If 2 of the bits were flipped during transmission, how many different possible strings could have been received?

E. What fraction of bytes (8-bit bit-strings) have at most 3 bits set?

Problem 8 (5 points). Suppose you have a n by k grid of positions. The bottom-left position is $(0,0)$ and top right is at position (n,k). If you can only move up 1 position or right 1 position, explain why the total possible number of paths is $\begin{pmatrix} n+k \\ n \end{pmatrix}$.

Problem 9 (10 points). Suppose you have a bag with n unique marbles and want show your friend k marbles and give them one of the marbles to play with.

A. First suppose, you choose k of the marbles, and your friend chooses 1 at random. Write an expression to count the number of ways to have chosen the k marbles and the one your friend will use.

B. Next suppose, your friend choses a marble at random from all n marbles, then you choose $k-1$ more marbles to show your friend. Write an expression to count the number of ways to have chosen the marble your friend will use and the $k-1$ other marbles you've shown your friend.

Problem 10 (10 points). Prove that the following quantities are equivalent for $0 \leq k \leq n$.

$$k \left(\begin{array}{c} n \\ k \end{array} \right) = n \left(\begin{array}{c} n-1 \\ k-1 \end{array} \right)$$