CSCI 246: Assignment 4
Due: March 13, 2026

Name: ____________________________

Problem 1 (4 points). For each of the below statements. Determine which of the following proof techniques would be most natural to prove the statement:

- Direct proof
- Proof by contrapositive
- Proof by contradiction
- Proof by induction

Note: You only need to pick a technique and justify why the technique is appropriate. No proof required.

A. If n^2 is even, then n is even.

B. For all n, $\sum_{k=1}^{n} 2k - 1 = n^2$.

C. There is no smallest positive real number.

D. The square-root of 2 is irrational.

Problem 2 (6 points). For each statement below, (i) write the statement symbolicaly and (ii) write its contrapositive.

A. If an integer is divisible by 6, then it is divisible by 3.

B. If n is prime, then n has no positive factors other than 1 and itself.

C. Let A and B be sets. A is a subset of B and has the same cardinality as B if and only if A is B.

Problem 3 (5 points). Consider the statement $P(n)$ (a predicate over the natural number $n \in \mathbb{N}$). Write down the proof template for how one would prove $P(n)$ holds for all natural numbers using proof by smallest counter-example.

Problem 4 (5 points). Let $a, b \in \mathbb{Q}$ be rational numbers with $a \neq 0$, prove that the equation $ax + b = 0$ has a unique solution (i.e., a unique value $c \in \mathbb{Q}$ such that $x = c$ makes the equation true).

Problem 5 (5 points). Let $a, b, c \in \mathbb{Z}$ be integers. Prove that if a does not divide bc, a does not divide b and a does not divide c.

Problem 6 (5 points). Let $n \in \mathbb{Z}$ be an integer. Prove that if n is divisible by 4, then n is even.

Problem 7 (5 points). Prove that $7^n - 1$ is divisible by 6 for any natural number $n \in \mathbb{N}_0$ (i.e., $n \geq 0$).

Problem 8 (5 points). Suppose you and a friend release an app. Initially, you and your friend are the only users. However, after running for several weeks you notice a pattern in which each week you have twice as many ***new*** users as you had the previous week, but each week 3 users delete thier account. Create a recurrence relation that models the number of users for week n. Solve the recurrence relation to get a closed-form solution (recurrence-free formula) that represents the number of users for week n. Then use the closed form solution to predict how many users you will have after 1 year if the pattern stays consistent (i.e., $n = 52$).

Problem 9 (10 points). A distributed system recieves requests over time. Let a_n be the number of requests handled during hour n. Two factors influence the number of requests handled:

1. Increased demand causes the number of requests in the current hour to grow to 11 times the number of requests handled in the previous hour.
2. At the same time, the systems caching mechanism serves requests for data that was frequently accessed in the past. This reduces the number of requests handled by 28 times the number of requests from two hours earlier.

In the first hour of operation, the system handled a single request. In the second hour, the system received 11 requests. Write a recurrence relation that models the number of requests handled at hour n. Solve the recurrence relation to get a closed form for a_n and then use the closed form solution to find the number of requests handle after 10 hours of operation.

Problem 10 (10 points). Suppose you are working in a biology lab studying bacteria growth. The number of bacteria b_n in hour n depends on reproduction and competition for nutrients.

1. Each hour, the backteria reproduce, producing six times the population from the previous hour
2. As nutrients become scarce, bacteria die off due to competition. This reduces the population proportionally by nine times the population from two hours earlier.

You started with two bacteria initially, after 1 hour you observed 9 bacteria. Write a recurrence relation capturing the population growth model (i.e., a recurrence relation for b_n). Then solve the recurrence relation to find a closed-form and use the closed form solution to determine the number of backteria after 8 hours of observation.