CSCI 246: Assignment 6
Due: April 17, 2026

Name: ____________________________

Problem 1 (6 points). Suppose you are designing a system that handles requests and returns one of the following results: Success (S), Timeout (T), or Error (E). Empirically, you determine that the system returns success 80% of the time, timeout 15% of the time, and error 5% of the time. Suppose you have two independent requests that are processes sequentially.

A. What is the sample space of the results returned by the two requests?

B. Let A be the event "at least one request is a success." List all outcomes in A.

C. What is the probability of the event A?

D. Let B be the event "both requests have the same output." List all outcomes in B.

E. What is the probability of the event B?

F. What is the probability of events A and B both happening?

Problem 2 (6 points). Suppose you are designing a system that requires logins to use passwords with 4 lowercase letters (a-z).

A. Assuming that passwords are uniformaly distributed. What is the sample space of all passwords?

B. What is the probability that a password consists of distinct letters?

C. What is the probability that a password contains at least one repeated letter?

Problem 3 (4 points). Consider a dataset that consists of 1000 users' information. Each user uses feature A or B (or both). You are given a summary that 600 users use feature A. 400 user feature B, and 250 use both A and B.

A. What is $P(A)$, $P(B)$, and $P(A \cap B)$?

B. Compute $P(A|B)$.

C. Compute $P(B|A)$.

D. Are events A and B independent? Explain why or why not.

Problem 4 (4 points). Let A and B be events with $P(A) = 0.4$ and $P(B) = 0.5$.

A. If A and B are independent, compute $P(A \cap B)$.

B. If A and B are disjoint, compute $P(A \cap B)$.

C. Can two nonzero-probability events be both independent and disjoint? Explain why or why not.

Problem 5 (10 points). 3 fair six-sided dice are rolled (rolls are independent).

A. Define a random variable X that is the product of the values on each die.

B. List the probability distribution of X.

C. Compute $E(X)$.

D. Compute $\text{Var}(X)$.

Problem 6 (10 points). A randomized algorithm processes 3 independent tasks. Each task succeeds with probability 0.8. Let X be the number of successful tasks.

A. Express X as a sum of indicator random variables (e.g., over $S_i = 0$ is task i failed, and $S_i = 1$ is task i succeeded).

B. Compute $E(X)$ using linearity of expectation.

C. What is the probability that all 3 tasks succeed?

Problem 7 (10 points). Let X be a random variable with: $P(X = 0) = 0.5$, $P(X = 1) = 0.3$, and $P(x = 3) = 0.2$.

A. Compute $E(X)$.

B. Compute $E(X^2)$.

C. Compute $\text{Var}(X)$.

Problem 8 (10 points). A spam filter flags emails as spam or not. Suppose 10% of all emails are spam, the filter correctly flags spam 95% of the time, and the filter incorrectly flags non-spam as spam 5% of the time.

A. If an email is flagged as spam, what is the probability that it is actually spam?

B. If an email is not flagged as spam, what is the probability that the email is actually spam?