Boolean Algebra: Group Exercises

CSCI 246

January 28, 2026

Problem 1. For each property, give an equation showing the property.

Example. *Commutative property* $\wedge$: $x \wedge y = y \wedge x$.

A. *Commutative property* $\vee$: $x \vee y = y \vee x$.

B. *Associativity* $\wedge$: $x \wedge (y \wedge z) = (x \wedge y) \wedge z$.

C. *Associativity* $\vee$: $x \vee (y \vee z) = (x \vee y) \vee z$.

D. *Identity element* $\wedge$: $1 \wedge x = x \wedge 1 = x$.

E. *Identity element* $\vee$: $0 \vee x = x \vee 0 = x$.

F. *Double negation:* $\neg\neg x = x$.

G. *DeMorgan's Law:* $\neg(x \wedge y) = \neg x \vee \neg y$ and $\neg(x \vee y) = \neg x \wedge \neg y$.

H. *Idempotency* $\vee$: $x \vee x = x$.

I. *Idempotency* $\wedge$: $x \wedge x = x$.

J. *Distributivity:* $(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$ and $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$.

K. *Law of excluded middle:* $x \vee \neg x = 1$

L. *Complement law:* $x \wedge \neg x = 0$.

Problem 2. Show that $a \implies b$ is equivalent to $(\neg a) \vee b$.

$\mathbf{a}$	$\mathbf{b}$	$\mathbf{a \implies b}$	$\mathbf{\neg a \vee b}$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

Problem 3. Prove that $(x \wedge y) \vee (x \wedge \neg y)$ is equivalent to x.

x	**y**	$(\mathbf{x} \wedge \mathbf{y}) \vee (\mathbf{x} \wedge \neg \mathbf{y})$
0	0	0
0	1	0
1	0	1
1	1	1

Alternatively, you can use the rules from Promblem 1 to prove the equivalence.

$$(x \wedge y) \vee (x \wedge \neg y)$$

By distributivity of $\vee$ and $\wedge$ we can foil the expression to:

$$(x \vee x) \wedge (x \vee \neg y) \wedge (y \vee x) \wedge (y \vee \neq y)$$

We can then simplify using idempotency of $\vee$ and law of excluded middle:

$$x \wedge (x \vee \neg y) \wedge (y \vee x) \wedge 1$$

Further simplify by multiplication identity, commutativity, and distributivity:

$$x \wedge (x \vee (\neg y \wedge y))$$

We next use the complement law and simplify using disjunctive identity:

$$x \wedge x$$

Finally, we simplify using idempotency of $\wedge$:

$$x$$

Problem 4. Show that the following are *tautologies*.

A. $(x \vee y) \implies (x \vee (y \wedge z)) \vee (x \vee \neg z)$.

$\mathbf{x}$	$\mathbf{y}$	$\mathbf{z}$	$(\mathbf{x} \vee \mathbf{y}) \implies (\mathbf{x} \vee (\mathbf{y} \wedge \mathbf{z})) \vee (\mathbf{x} \vee \neg \mathbf{z})$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

B. $(x \wedge (y \implies z)) \implies ((x \wedge y) \implies (x \wedge z))$.

$\mathbf{x}$	$\mathbf{y}$	$\mathbf{z}$	$(\mathbf{x} \wedge (\mathbf{y} \implies \mathbf{z})) \implies ((\mathbf{x} \wedge \mathbf{y}) \implies (\mathbf{x} \wedge \mathbf{z}))$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

C. $(x \vee y) \wedge (\neg x \vee z) \implies (y \vee z)$.

$\mathbf{x}$	$\mathbf{y}$	$\mathbf{z}$	$(\mathbf{x} \vee \mathbf{y}) \wedge (\neg \mathbf{x} \vee \mathbf{z}) \implies (\mathbf{y} \vee \mathbf{z})$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

D. $x \vee (\neg x \wedge (x \vee y)) \vee \neg y$.

$\mathbf{x}$	$\mathbf{y}$	$\mathbf{x} \vee (\neg \mathbf{x} \wedge (\mathbf{x} \vee \mathbf{y})) \vee \neg \mathbf{y}$
0	0	1
0	1	1
1	0	1
1	1	1