Proof Systems: Group Exercises

CSCI 246

January 30, 2026

Problem 1. Prove the following proposition: $\vdash A \to (A \wedge A)$.

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}\,id \qquad \dfrac{}{A \vdash A}\,id}{A \vdash A \wedge A}\,\wedge R}{\vdash A \to (A \wedge A)}\,\to R$$

Problem 2. Prove the following proposition: $\vdash (A \wedge B) \to A$.

$$\dfrac{\dfrac{\dfrac{}{A, B \vdash A}\,id}{A \wedge B \vdash A}\,\wedge L}{\vdash (A \wedge B) \to A}\,\to R$$

Problem 3. Prove the following proposition: $\vdash A \to (A \vee B)$.

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}\,id}{A \vdash A \vee B}\,\vee R_1}{\vdash A \to (A \vee B)}\,\to R$$

Problem 4. Prove the following proposition: $\vdash (A \wedge B) \to (B \wedge A)$.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, B \vdash A}\,id \qquad \dfrac{}{A, B \vdash B}\,id}{A, B \vdash B \wedge A}\,\wedge R}{A \wedge B \vdash B \wedge A}\,\wedge L}{\vdash (A \wedge B) \to (B \wedge A)}\,\to R$$

Problem 5. Prove the following proposition: $\vdash (A \vee B) \to (B \vee A)$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,id}{A \vdash B \vee A}\,\vee R_2 \qquad \dfrac{\dfrac{}{B \vdash B}\,id}{B \vdash B \vee A}\,\vee R_1}{A \vee B \vdash B \vee A}\,\vee L}{\vdash (A \vee B) \to (B \vee A)}\,\to R$$

Problem 6. Prove the following proposition: $\vdash \neg(A \wedge \neg A)$.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,id}{A, \neg A \vdash}\,\neg L}{A \wedge \neg A \vdash}\,\wedge L}{\vdash \neg(A \wedge \neg A)}\,\neg R$$

Problem 7. Prove the following proposition: $\vdash A \vee \neg A$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,id}{\vdash A, \neg A}\,\neg R}{\vdash A \vee \neg A, \neg A}\,\vee R_1}{\vdash A \vee \neg A, A \vee \neg A}\,\vee R_2}{\vdash A \vee \neg A}\,CR$$

Problem 8. Prove the following proposition: $\vdash \neg\neg A \to A$.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,id}{\vdash A, \neg A}\,\neg R}{\neg\neg A \vdash A}\,\neg L}{\vdash \neg\neg A \to A}\,\to R$$

Problem 9. Prove the following proposition: $\vdash ((A \to B) \to A) \to A$.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A, B}\,id}{\vdash A, A \to B}\,\to R \qquad \dfrac{}{A \vdash A}\,id}{(A \to B) \to A \vdash A}\,\to L}{\vdash ((A \to B) \to A) \to A}\,\to R$$

Problem 10. Prove the following proposition: $\vdash ((A \to B) \wedge (B \to C)) \to (A \to C)$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A, B, C}\,id \qquad \dfrac{}{B \vdash B, C}\,id}{A, A \to B \vdash B, C}\,\to L \qquad \dfrac{}{C \vdash C}\,id}{A, A \to B, B \to C \vdash C}\,\to L}{A \to B, B \to C \vdash A \to C}\,\to R}{(A \to B) \wedge (B \to C) \vdash A \to C}\,\wedge L}{\vdash ((A \to B) \wedge (B \to C)) \to (A \to C)}\,\to R$$

Problem 11. Prove the following proposition: $A \to (B \to C) \vdash (A \wedge B) \to C$.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, B \vdash A, C}\,id \qquad \dfrac{\dfrac{}{A, B \vdash B, C}\,id \qquad \dfrac{}{A, B, C \vdash C}\,id}{A, B, B \to C \vdash C}\,\to L}{A, B, A \to (B \to C) \vdash C}\,\to L}{A \wedge B, A \to (B \to C) \vdash C}\,\wedge L}{A \to (B \to C) \vdash (A \wedge B) \to C}\,\to R$$

Problem 12. Prove the following proposition: $(A \wedge B) \to C \vdash A \to (B \to C)$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B \vdash C, A}\,id \qquad \dfrac{}{A, B \vdash C, B}\,id}{A, B \vdash C, A \wedge B}\,\wedge R \qquad \dfrac{}{A, B, C \vdash C}\,id}{A, B, (A \wedge B) \to C \vdash C}\,\to L}{A, (A \wedge B) \to C \vdash B \to C}\,\to R}{(A \wedge B) \to C \vdash A \to (B \to C)}\,\to R$$