Lists: Group Exercises

CSCI 246

February 2, 2026

Problem 1. Consider the set of elements $A = \{a, b, c\}$. .

A. Write all lists of length 1 using elements in A:

$$(a) \qquad (b) \qquad (c)$$

B. Write all ordered pairs using elements of A:

$$(a,a) \qquad (a,b) \qquad (a,c) \qquad (b,a) \qquad (b,b) \qquad (b,c) \qquad (c,a) \qquad (c,b) \qquad (c,c)$$

C. How many lists are there that contain no repeated elements of A?

There are 16 lists with no repeated elements:
1 of length 0, 3 of length 1, 6 of length 2, and 6 of length 3.

Problem 2. Binary numbers are lists that contain only the elements 0 and 1.

A. Write all binary numbers of length 5 that are a palandrome (written the same forwards and backwards).

00000 00100 01010 01110 10001 10101 11011 11111

B. How many binary numbers of length 6 have an equal number of 0s and 1s?

There are 20 binary numbers of length 6 with an equal number of 0s and 1s. Any valid binary number of length 6 is determined by the set of positions where 1s appear (all others positions are 0). Thus we can answer by counting the number of subsets of $\{1, 2, 3, 4, 5, 6\}$ have length 3.

C. How many binary numbers of length 10 or less have more 1s than 0s?

There are 848 binary numbers of length 10 or less that have more 1s than 0s. For odd length numbers exactly half of the binary numbers of that length have more 1s than 0s. For even numbers, there are numbers that have an equal number of 1s and 0s; of the remaining numbers half have more 1s and 0s.

For odd lengths (1, 3, 5, 7, 9) there are $1 + 4 + 16 + 64 + 256 = 341$ binary numbers with more 1s than 0s.

For even lengths (0, 2, 4, 6 , 8, 10) there are $0 + 1 + 5 + 22 + 93 + 386 = 507$ binary numbers with more 1s than 0s.

Problem 3. Let A be a set of $n \in \mathbb{N}$ elements.

A. How many lists of length k are there?

There are n^k lists of length k.

B. How many lists without repeated elements of length k are there?

There are a total of $(n)_k = n * (n-1) * ... * (n-k)$ lists of length k with no repeats.

C. How many lists of length k have at least 1 repetitions?

There are a total of $n^k - (n)_k$ lists with at least 1 repetitions.

Problem 4. Let $A = \{a, b, c, d\}$ be a list of elements.

A. How many lists of length 4 can be formed using elements of A?

There are a total of $4^4 = 256$ lists of length 4.

B. How many lists of length 4 start with a?

There are a total of $1 * 3^4 = 64$ lists of length 4 that start with a.

C. How many lists of length 4 start and end with the same element?

There are a total of $4 * 4 * 4 * 1 = 3^4 = 64$ lists of length 4 with the same first and last element.

Problem 5. Let $A = \{1, 2, 3, 4, 5\}$ be a list of elements.

A. How many lists of length 5 can be formed from elements of A?

There are a total of $5^5 = 3125$ lists of length 5.

B. How many lists of length 5 contain no repeated elements?

There are a total of $(5)_5 = 5! = 5 * 4 * 3 * 2 * 1 = 120$ lists of length 5 with no repeated elements.

C. How many lists of length 5 contain at least one repeated element?

There are a total of $5^5 - (5)_5 = 3125 - 120 = 3005$ lists of length 5 with at least one repeated element.

Problem 6. Recall that binary numbers are lists using only 0 and 1.

A. How many binary numbers of length 7 are palindromes?

There are a total of 16 binary numbers of length 7 that are palindromes.

B. How many binary numbers of length 8 are not palindromes?

There are a total of 16 binary numbers of length 8 that are palindromes.

C. Explain why the answers to part **A** and **B** are equal.

In odd length numbers the middle bit can be either 0 or 1; similarly, in even length numbers the middle two bits can either be 0 or 1 but must be equal (i.e., there is still only 2 options for the middle bits to take).