Sets: Group Exercises

CSCI 246

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Problem 1. Consider the sets $A = \{1, \{1\}, \{1, 2\}\}$ and $B = \{1, 2\}$. Determine whether each state is true or false.

A. $1 \in A$. true because 1 is an element of $A = \{1, \{1\}, \{1, 2\}\}$.

B. $\{1\} \in A$. true because $\{1\}$ is an element of $A = \{1, \{1\}, \{1, 2\}\}$

C. $\{1\} \subseteq A$. true because 1 is an element of $A = \{1, \{1\}, \{1, 2\}\}$.

D. $\{1\} \in B$, false because $\{1\}$ is not an element of B.

E. $\{1\} \subseteq B$. true because 1 appears in $B = \{1, 2\}$.

F. $\{1, 2\} \in A$. true because $\{1, 2\}$ appears in $A = \{1, \{1\}, \{1, 2\}\}$.

G. $\{1, 2\} \subseteq A$. false because 2 is not an element of A.

Problem 2. Translate each description into *set-builder notation*, then list the set of elements explicitly.

A. The set of integers whose square is less than 20.

$$\{x \in \mathbb{Z} : x^2 \leq 20\} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

B. The set of natural numbers that divide 144.

$$\{x \in \mathbb{N} : x | 144\} = \{1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144\}$$

C. The set of binary numbers with exactly two 1s and up to 4 0s. You may assume $count(b, 1)$ counts the number of 1s appearing in the binary number b and similarly $count(b, 0)$ counts the number of 0s.

$$\{x \text{ is a binary number} : count(x, 1) = 2 \text{ and } count(x, 0) \leq 4\}$$

$$\left\{ \begin{array}{c} 11, \\ 011, 101, 110, \\ 0011, 0101, 0110, 1001, 1010, 1100, \\ 00011, 00101, 00110, 01001, 01010, 01100, 10001, 10010, 10100, 11000, \\ 000011, 000101, 000110, 001001, 001010, 001100, 010001, 010010, \\ 010100, 011000, 100001, 100010, 100100, 101000, 110000 \end{array} \right\}$$

Problem 3. Describe the set defined by the following set-builder notations.

A. $\{x \in \mathbb{Z} : x \equiv 1(mod 3)\}$.

The set of integer numbers that when divided by 3 have a remainder of 1.

B. $\{x \in \mathbb{Z} : 1 < x$ and there is no $y \in Z$ s.t. $1 < y < x$ and $y|x\}$

The set of prime numbers.

C. $\{x \in \mathbb{Z} : 0 \leq x\}$

The whole numbers

Problem 4. Let $A = \{a, b, c, d\}$.

A. List all subsets of A.

$$\begin{array}{c} \emptyset \\ \{a\} \quad \{b\} \quad \{c\} \quad \{d\} \\ \{a,b\} \quad \{a,c\} \quad \{a,d\} \quad \{b,c\} \quad \{b,d\} \quad \{c,d\} \\ \{a,b,c\} \quad \{a,b,d\} \quad \{a,c,d\} \quad \{b,c,d\} \\ \{a,b,c,d\} \end{array}$$

B. Count the number of subsets by *cardinality*.

Subsets of size 0: 1
Subsets of size 1: 4
Subsets of size 2: 6
Subsets of size 3: 4
Subsets of size 4: 1

Total number of subsets: 16

C. Write the powerset of A (i.e., 2^A / $\mathcal{P}(A)$).

$$\left\{ \begin{array}{c} \emptyset, \\ \{a\}, \quad \{b\}, \quad \{c\}, \quad \{d\}, \\ \{a,b\}, \quad \{a,c\}, \quad \{a,d\}, \quad \{b,c\}, \quad \{b,d\}, \quad \{c,d\}, \\ \{a,b,c\}, \quad \{a,b,d\}, \quad \{a,c,d\}, \quad \{b,c,d\}, \\ \{a,b,c,d\} \end{array} \right\}$$

D. What is the cardinality of $\mathcal{P}(A)$?

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Problem 5. For each statement, decide whether its true for all sets or give a counter-example.

A. if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. true

B. if $A \subset B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. true

C. $A \subseteq B$ and $B \subseteq A$ if and only if $A = B$. true

D. If $A \subset B$, then $|A| < |B|$. true

E. If $A \subseteq B$, then $|\mathcal{P}(A)| < |\mathcal{P}(B)|$. false. Consider $A = B = \emptyset$

Problem 6. Let $A = \{x \in Z : x \text{ is even}\}$ and $B = \{x \in Z : 4|x\}$. Prove $B \subseteq A$.

Proof. Let x be an arbitrary element of B. Since $x \in B$, we know $4|x$. Thus, by definition there is some $k \in Z$ such that $x = 4k$. Clearly, x is even (i.e., $x = 2(2k)$). Thus, $x \in A$. Since, we chose x arbitrarily, we know that every element of B is also an element of B. Thus, by definition B is a subset of A. □