Set Operations: Group Exercises

CSCI 246

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Problem 1. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 8\}$, and $c = \{5, 6, 7\}$. Compute the following:

A. $A \cup B = \{1, 2, 3, 4, 5, 8\}$.

B. $A \cap B = \{3, 4\}$.

C. $A - B = \{1, 2, 5\}$.

D. $B - A = \{8\}$.

E. $A \triangle B = \{1, 2, 5, 8\}$.

F. $(A \cup B) \cap C = \{5\}$.

G. $A \cap (B \cup C) = \{3, 4, 5\}$.

H. Are A and C disjoint? No

I. Are B and C disjoint? Yes

Problem 2. Let A, B, C be sets as defined in **Problem 1**.

A. Compute $|A| = 5$.

B. Compute $|B| = 3$.

C. Compute $|A \cap B| = 2$.

D. Compute $|A \cup B| = 6$.

E. Verify $|A \cup B| = |A| + |B| - |A \cap B|$. $\quad 6 = 5 + 3 - 2$.

F. Simplify the equation in part **E**, assuming that A and B are disjoint.

$$|A \cup B| = |A| + |B|$$

Problem 3. Let A and B be sets as defined in **Problem 1**.

A. Express $A \triangle B$ using only union, intersection, and set difference.

$$A \triangle B = (A \cup B) - (A \cap B)$$

B. Verify that $A \triangle B$ is equivalent to your answer to part **A**.

$$A \triangle B = \{1, 2, 5, 8\} = \{1, 2, 3, 4, 5, 8\} - \{3, 4\} = (A \cup B) - (A \cap B)$$

C. When is $A \triangle B = \emptyset$?

When $A = B$.

Problem 4. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 8\}$, and $c = \{5, 6, 7\}$. Compute the following:

A. Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

$$A \cap (B \cup C) = \{3, 4, 5\} = \{3, 4\} \cup \{5\} = (A \cap B) \cup (A \cap C)$$

B. Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

$$A \cup (B \cap C) = \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5\} \cup \emptyset = A \cup (B \cap C)$$

Problem 5. For arbitrary sets A, B, and C, answer whether the following statements are true or false.

A. $A - B = B - A$. false

B. If $A \cap B = \emptyset$, then $A \cup B = B \triangle A$. true

C. If $A \subseteq B$, then $A \cap C \subseteq B \cap C$. true

D. $A \cap (B - C) = (A \cap B) - C$. false

Problem 6. Prove for all sets A and B that $A \cup B = B \cup A$.

Proof. By definition $A \cup B = \{x : x \in A \vee x \in B\}$.
Since disjunction is commutative, we have $A \cup B = \{x : x \in B \vee x \in A\}$.
By definition definition $B \cup A = \{x : x \in B \vee x \in A\}$.
Clearly, $A \cup B = B \cup A$. □

Alternatively, you may prove in a similar fashion for all elements x, $x \in (A \cup B) \iff x \in (B \cup A)$.

Problem 7. Prove for all sets A and B that $A - (B \cup C) = (A - B) \cap (A - C)$.

Proof. By definition $A - (B \cup C) = \{x : x \in A \wedge x \notin (B \cup C)\}$.
Further simplifying we have, $A - (B \cup C) = \{x : x \in A \wedge \neg(x \in B \vee x \in C)\}$.
Simplifying, we have $A - (B \cup C) = \{x : (x \in A \wedge x \notin B) \wedge (x \in A \wedge x \notin C)\}$.
By definition of $\cap$, we have $A - (B \cup C) = \{x : x \in A \wedge x \notin B\} \cap \{x : x \in A \wedge x \notin C\}$.
By Definition, we $A - B = \{x : x \in A \wedge x \notin B\}$ and $A - C = \{x : x \in A \wedge x \notin C\}$.
Thus, we may conclude $A - (B \cup C) = (A - B) \cap (A - C)$. □

Problem 8. Prove for all sets A, B, and C that $A \cup (B \cup C) = (A \cup B) \cup C$.

Proof. By definition $A \cup (B \cup C) = \{x : x \in A \vee (x \in B \cup C)\}$.
Further expanding definitions and simplifying gives: $A \cup (B \cup C) = \{x : x \in A \vee (x \in B \vee x \in C)\}$.
By associativity of $\vee$, we have $A \cup (B \cup C) = \{x : (x \in A \vee x \in B) \vee x \in C\}$.
Folding some definitions, we have $A \cup (B \cup C) = \{x : x \in (A \cup B) \vee x \in C\} = (A \cup B) \cup C$. □